Oxford Cambridge and RSA

Thursday 22 May 2025 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes

QP

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

1 **(a)** A matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$.

Describe the transformation represented by $\mathbf{M}$. **[2]**

(b) Write down the 2×2 matrix that represents a rotation of 90° anticlockwise about the origin. **[1]**

(c) Write down the 3×3 matrix that represents a reflection in the x–z plane. **[1]**

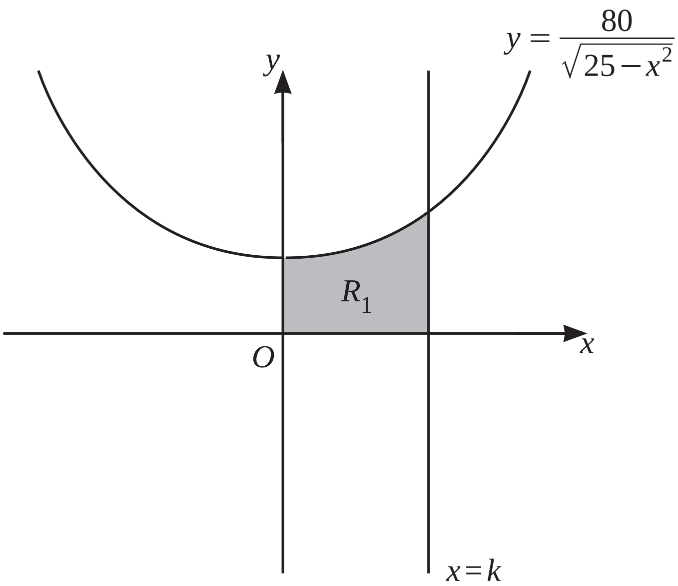
2 **(a)** Given that $y = \cosh^{-1}\left(\frac{1}{3}x\right)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. **[1]**

(b) Determine an equation of the normal to the curve $y = \cosh^{-1}\left(\frac{1}{3}x\right)$ at the point where $x = 5$.

Give your answer in the form $ax + by = c + \ln d$, where a, b, c and d are integers. **[4]**

3 The region R_1 is bounded by the curve $y = \dfrac{80}{\sqrt{25 - x^2}}$, the line $x = k$ and the coordinate axes, as shown in **Fig. 1**.

Fig. 1

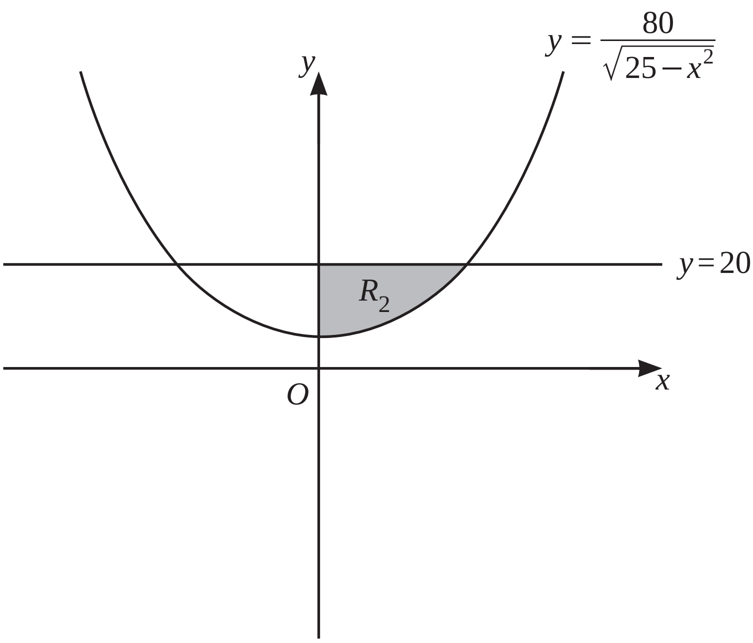


(a) **In this question you must show detailed reasoning.**

Given that the area of R_1 is $\dfrac{40}{3}\pi$, determine the value of k. **[3]**

The region R_2 is bounded by the curve $y = \dfrac{80}{\sqrt{25 - x^2}}$, the line $y = 20$ and the y-axis as shown in **Fig. 2**.

Fig. 2



(b) Find, in an exact form, the volume of the solid formed when R_2 is rotated by 2π radians about the ***y*-axis**. **[3]**

4 The equation $5x^3 - 4x^2 + 10 = 0$ has roots α, β and γ.

(a) Write down the values of $\alpha+\beta+\gamma$, $\alpha\beta+\beta\gamma+\gamma\alpha$ and $\alpha\beta\gamma$. **[2]**

(b) By expanding $(\alpha\beta+\beta\gamma+\gamma\alpha)^2$, determine the value of $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$. **[3]**

(c) By expanding a suitable expression, find the value of $\alpha^2+\beta^2+\gamma^2$. **[2]**

(d) Hence find a cubic equation with integer coefficients that has roots α^2, β^2, and γ^2. **[2]**

5 A vector equation of the plane Π_1 is $\mathbf{r} = \begin{pmatrix}1\\4\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\-1\end{pmatrix} + \mu\begin{pmatrix}1\\1\\0\end{pmatrix}$.

(a) **Verify** that a cartesian equation of Π_1 is $x - y + 2z = 3$. **[1]**

For some real constant a, cartesian equations of planes Π_2 and Π_3 are

$$\begin{aligned} \Pi_2 &: \quad x \qquad\quad - 3z = 1 \\ \Pi_3 &: \quad ax - y - z = 4 \end{aligned}$$

(b) By considering a suitable matrix, show that Π_1, Π_2 and Π_3 intersect at a single point for all values of a except $a = 2$. **[3]**

(c) Use the matrix from part **(b)** to find the coordinates of the point of intersection of Π_1, Π_2 and Π_3 in the case where $a = 3$. **[2]**

(d) In the case where $a = 2$, determine the geometrical arrangement of Π_1, Π_2 and Π_3. **[2]**

6 **In this question you must show detailed reasoning.**

The series S_n is given by $S_n = \left(\frac{1}{5} \times \frac{1}{15}\right) + \left(\frac{1}{15} \times \frac{1}{25}\right) + \cdots + \left(\frac{1}{10n-5} \times \frac{1}{10n+5}\right)$ for $n \in \mathbb{Z}^+$.

(a) Use the method of differences to show that, for all $n \in \mathbb{Z}^+$, $S_n < \frac{1}{50}$. **[5]**

Let $S_\infty = \lim\limits_{n\to\infty} S_n$.

(b) Given that, for some value of k, $S_\infty = \frac{1}{2450} + S_k$, find the value of k. **[3]**

7 A 3-D coordinate system, whose units are metres, is set up to model a street containing telephone cables T_1 and T_2.

The cables are modelled as straight lines with vector equations

$$T_1\colon\ \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \text{ and } T_2\colon\ \mathbf{r} = \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.$$

(a) Show that the cables do not intersect. **[3]**

To access the cables for maintenance, a ladder can be used. The base of the ladder is placed at a fixed point on the ground.

The ladder is modelled as a straight-line segment. The base of the ladder is modelled as being located at the point (4, 5, 0).

(b) Determine the minimum length of the ladder required so that it reaches cable T_1. Give your answer in centimetres to the nearest centimetre. **[4]**

(c) Identify a modelling assumption used in part **(a)** that is unrealistic, **and** which could affect your answer to this part. **[1]**

8 **In this question you must show detailed reasoning.**

In this question, arguments of complex numbers are in the interval $[0, 2\pi)$.

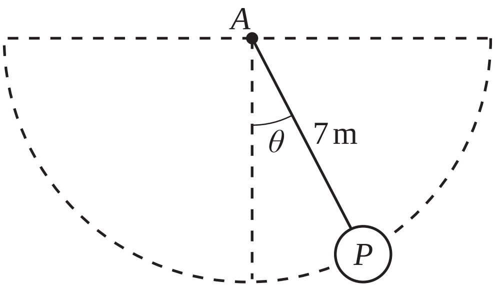
(a) Given that $z = -5 + 5\mathrm{i}$, express z in exact exponential form. **[2]**

(b) Given that $w = \dfrac{36\cos\left(\frac{1}{7}\pi\right) - 36\mathrm{i}\sin\left(\frac{1}{7}\pi\right)}{27\sin\left(\frac{3}{7}\pi\right) + 27\mathrm{i}\cos\left(\frac{3}{7}\pi\right)}$, express w in exact exponential form. **[5]**

9 A pendulum comprises an object P of mass m kg and a string of length 7 m. One end of the string is attached to P and the other end is attached to a fixed point A.

At time t seconds, $t \geqslant 0$, the string forms an angle of θ radians, measured anti-clockwise, from the downward vertical through A, as shown in the diagram.

When $t = 0$, $\theta = \theta_0 > 0$ and P is released from rest. You may assume that in the subsequent motion P moves along the arc of a circle, centre A and radius 7 m, and that $|\theta| \leqslant \theta_0$ for all $t \geqslant 0$.



The motion of P is modelled by the following differential equation.

$$\frac{d^2\theta}{dt^2} + \frac{7}{5}\sin\theta = 0 \ (*)$$

In some situations, it is appropriate to approximate $(*)$ with the following differential equation.

$$\frac{d^2\theta}{dt^2} + \frac{7}{5}\theta = 0 \ (**)$$

(a) Explain why it would be appropriate to model the motion of P with the differential equation $(**)$ when $\theta_0 = \frac{1}{15}\pi$ but not when $\theta_0 = \frac{1}{3}\pi$. **[1]**

You are now given that $\theta_0 = \frac{1}{15}\pi$.

(b) By finding the particular solution to the differential equation $(**)$, determine the total **distance** travelled by P in the first 6 seconds of the motion according to $(**)$. **[6]**

An additional force now acts on P. It can be shown that it is now appropriate to model the motion of P with the differential equation $\frac{d^2\theta}{dt^2} + \frac{k}{m}\frac{d\theta}{dt} + \frac{7}{5}\theta = 0$ where $k > 0$.

(c) Find the range of values of m, in terms of k, for which the motion of the pendulum is overdamped. **[2]**

10 In this question you must show detailed reasoning.

(a) Use de Moivre's Theorem to show that, if $\cos 5\theta \neq 0$,

$$\tan 5\theta \equiv \frac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^2\theta + 1}.$$ **[4]**

(b) **(i)** By considering the equation $\tan 5\theta = 1$, use the result in part **(a)** to find the exact roots of the equation

$$t^4 - 4t^3 - 14t^2 - 4t + 1 = 0.$$

Give the roots in the form $t = \tan\phi$ where $0 < \phi < \pi$. **[4]**

(ii) By first expressing $t^4 - 4t^3 - 14t^2 - 4t + 1 = 0$ in the form $(t-1)^4 = kt^2$, where k is a constant to be determined, show that

$$\tan\left(\frac{9}{20}\pi\right) = 1 + \sqrt{5} + \sqrt{5 + 2\sqrt{5}}.$$ **[3]**

END OF QUESTION PAPER